Development of a closed-form 3-D RBS beam finite element and associated case studies

Samuel Kinde Kassegne*

Department of Mechanical Engineering, College of Engineering, San Diego State University, 5500 Campanile Drive, San Diego, CA 92182-1323, United States

Received 15 January 2006; received in revised form 9 September 2006; accepted 11 September 2006

Abstract

The 1994 Northridge and the 1995 Kobe earthquakes demonstrated that steel moment resisting frames (SMRF) which were traditionally considered to provide an ideal structural system in resisting lateral earthquake loads could indeed suffer brittle fractures at the beam–column connections. To remedy this, competing connection design improvements have been adopted, among which is the radius-cut Reduced Beam Section (RBS) moment connection. As the design of a vast majority of these SMRF buildings is drift-controlled, the reduction in lateral stiffness and hence the increase in drifts in general 3-dimensional SMRF buildings with RBS moment connections – particularly those with torsional irregularity – is investigated here through development and verification of a new closed-form 3-D RBS finite element based on Timoshenko’s shear deformation theory for beams. The study shows that the increase in story drifts in 3-dimensional torsionally irregular SMRF buildings with RBS moment connections could be as high as 15% under the action of lateral and gravity loads. This is in contrast to the current recommendation of increasing elastic story drifts by a maximum of 9%–10% for SMRF buildings to account for stiffness reduction in the presence of RBS connections. The study also shows that the decrease in weak axis joint rotational and shear stiffness coefficients of beams with RBS connections on both sides – and unsupported laterally – could be as high as 67% for flange area reductions of only 30%.

Keywords: Earthquake design; RBS sections; Frames; Connections; Drift; Irregular buildings; Stiffness; Earthquakes

1. Introduction

One of the most important outcomes of the structural investigations of damages in the Northridge earthquake of 1994 was the finding of brittle fracture failures in welds at beam–column connections in as many as 200 steel special moment resisting frame (SMRF) buildings in the Los Angeles area [24]. While no actual collapse of steel moment frame buildings occurred, greater damage and even total collapse of these buildings could have resulted if the ground-shaking had continued for a longer time. The 1995 Kobe earthquake that hit the city of Kobe, Japan, a year after the Northridge earthquake had much more adverse consequences where a number of steel moment frame buildings suffered partial or total collapse, again confirming the fact that steel moment connections were not as safe as they were previously believed to be.

As a direct consequences of these damages, a number of initiatives were forwarded to investigate the causes for the poor performance of moment connections and to come up with improved seismic design and retrofitting techniques for steel moment frames. Out of these initiatives, a number of competing ‘prequalified’ design improvements in beam column connections in special steel moment resisting frames have found their way to the design practice. One of the design improvements uses what is called Reduced Beam Section (RBS) or dog-bone moment connection and is a prequalified connection [7] in which a portion of the top and bottom flanges is cut out at some distance from the face of columns. The RBS connection is essentially intended to pull the plastic hinge away from the face of the column where the stress and strain demands may cause the weld to fracture.

In the past several years, a number of steel buildings with SMRF have increasingly made use of ‘prequalified’ RBS moment connections. A number of structural engineering design firms in the US and Pacific Rim countries now design...
such SMRF frames with RBS moment connections on a routine basis. The typical modelling of the effect of RBS moment connections on drift for routine analysis and design has been based on the ‘10% drift increase’ approach recommended by Grubbs [10], Moore et al. [16], and the ‘9% drift increase for 50% cut’ approach recommended by FEMA [7] and SAC [18]. The ‘10%’ limit proposed by Moore et al. [16] was based on a 2-dimensional analysis results reported by Grubbs [10] which do not take into account diaphragm rotations, weak axis bending, stiffness irregularities and a combination of arbitrary gravity and lateral loads. However, there is no adequate information on what study the ‘9%’ limit (for 50% radius cut) proposed by the FEMA guideline is based on. Further, the SEAOC Seismology Committee commentary and recommendations on FEMA 350 is quiet on the subject [19]. Similarly, there does not seem to be any published reference where AISC or any other code bodies question the issue. It seems, therefore, that a significant number of moment resisting frames with RBS moment connections are currently being analyzed based on gross geometry and then story displacements adjusted for stiffness reduction by a flat 9% or 10%. In a further note, the typical tests on RBS systems such as those done by [22,5,4] were based on subassemblages which consisted of typically W36 beams and W14 columns and were 2-dimensional in nature and, consequently, did not shed light on elastic diaphragm torsional rotations, weak axis bending of beams and the interaction of gravity and lateral loads.

However, the past few years have witnessed an ongoing research to develop rational, accurate and efficient methods for the exact analysis of SMRF with RBS connections. Hailu et al. [11] had reported a preliminary work that employs the use of mixed elements (shell and beam elements) constrained with MPC – multi-point constraints – to model the actual geometry and stiffness reductions in RBS systems. The method exploits the advantages of solid and shell elements for accuracy and beam elements for economy. Numerical results for realistic building configurations have not been reported yet, however. Further, the computational time and modelling effort required in this approach may turn out to be too prohibitive for routine design purposes. The work of Chambers [2] is the first attempt on closed-form solution for moment frames with RBS connection. Chambers [2] predicted a maximum drift increase of 10.6% for a 2-dimensional frame system with RBS connection of 40% flange reductions. They also reported a maximum decrease of 15.1% in the stiffness coefficient corresponding to joint rotations in frames with RBS connections of 44% reduction in flange area. Their work is, however, limited to two-dimensional frames and is based on classical beam theory which underestimates deflections both in beams and columns. Further, the effect of gravity loads, shear deformations, torsion and minor axis bending were not included in their formulations.

To date, therefore, no report exists in the open literature on a complete 3-dimensional frame system analysis – including shear deformation – using exact FEA modelling of RBS members. In the absence of such studies, the effect of 3-dimensional loading, shear deformation, weak axis bending and most importantly torsion and a combination of gravity and lateral loads – particularly for moment frames with irregular stiffness distribution – were at best unknown for routine design purposes.

In this work, this need is addressed by developing a new and numerically exact stiffness matrix for RBS members based on the potential energy approach for deriving stiffness coefficients. The stiffness coefficients developed here are for a general 3D beam element and include major and minor axis bending, shear deformation and torsion and are based on Timoshenko’s Beam Theory. A simple programme that uses the newly derived RBS stiffness matrix was developed in the course of this research. The only additional input required for each beam with RBS section are the parameters ‘\(a\)’, ‘\(b\)’ and ‘\(c\)’ as shown in Fig. 1. As the methodology is based on the exact numerical integration of stiffness coefficients just like the ordinary 3D beam stiffness matrix, increase in computational time and effort is nonexistent. Further, the evaluation of fixed end forces, member forces and beam span deflection remain virtually identical. This, therefore, demonstrated that the stiffness coefficients can be directly and seamlessly integrated to existing structural engineering software without affecting programme architecture and performance.

### 2. Formulations

The works of Hailu et al. [11] which used MPC – multi-point constraints – to model the actual geometry and stiffness reductions in RBS systems and, in particular, that of [2], which is the first published attempt on closed-form solution for modelling frames with RBS connections serve as starting points for the current formulations.

### Notations

- \(b_f\): width of beam flange;
- \(t_f\): thickness of beam flange;
- \(d\): depth of beam;
- \(L\): total length of beam;
- \(I_{maj}\): major moment of inertia;
- \(I_{min}\): minor moment of inertia;
- \(A\): area of cross section;
- \(A_{smax}\): major shear area;
- \(A_{smin}\): minor shear area;
- \(J\): polar moment of inertia;
- \(W\): virtual work done by a unit load;
- \(R\): radius of flange cut;
- \(r\): ratio of reduced flange width to the original flange width;
- \(a\): distance from face of column to beginning of radius cut;
- \(b\): half-width of radius cut;
- \(c\): maximum depth of radius cut;
- \(E\): Young’s modulus;
- \(G\): shear modulus.
A 12-degrees of freedom beam element with flange reductions as shown in Fig. 1 and nodal displacements and joint forces shown in Fig. 2 is considered. For simplicity and clarity, the particular case of only end ‘1’ deforming while end ‘2’ is kept fixed is considered here. This approach will help determine the diagonal stiffness sub-matrices \([K_{11}] \) and \([K_{22}] \) indicated in Eq. (7.a). The coupling (off-diagonal) sub-matrices \([K_{12}] \) and \([K_{21}] \) will be determined through equilibrium equations and symmetry considerations, respectively. In standard stiffness-based finite element (FE) methodology, there are a number of approaches available for deriving the stiffness coefficients like the minimization of potential energy, virtual work method, and principle of variational calculus. In this case, the virtual work method is used where expressions for the virtual works done in each of the directions of the six degrees of freedom (per node) of a 3D beam element with RBS moment connections are derived. Some of the lengthy expressions, which correspond to the 2D response such as major axis bending, were already reported by Chambers [2]. However, the presence of shear deformation in the current formulation introduces significant additional terms to those reported by Chambers [2]. These additional terms in the FE formulation for 2D responses are, therefore, presented here along with the new 3D expressions that correspond to torsion and more importantly minor axis bending.

2.1. Virtual work in axial direction

The internal work done by a unit axial load applied at end ‘1’ of a beam element with RBS is:

\[
W_{\text{int, axial}} = 2 \int_0^a \frac{f_1}{E} \frac{dx}{A} + \int_0^{Le} \frac{f_1}{E} \frac{dx}{A} + 2 \int_{-b}^{b} \frac{f_1}{E} \frac{dx}{A}[1 - kn_{\text{axial}}(\sqrt{R^2 - x^2} + R^1)] \tag{1.a}
\]

where \(n_{\text{axial}} = \frac{4t}{R}, E — \text{Young’s modulus, } A — \text{area of cross section, } R — \text{the radius of flange-cut, and } t_f — \text{thickness of flange of beam. Parameters } Le, a, b \text{ and } R^1 \text{ are defined in the Appendix.} \ k = 4 \text{ for flange reductions both in the bottom and top parts of the beam; otherwise, } k = 2 \text{ for the case of flange reductions only at the bottom. The joint force, } f_1 \text{ is shown in Fig. 2 along with the other joint forces.}

Integrating Eq. (1.a) using Mathematica software [15] or alternatively the trigonometric substitutions adopted by Chambers [2],

\[
W_{\text{int, axial}} = \frac{f_1}{EA} \left[ 2a + Le + \frac{4}{n_{\text{axial}}} (2\Omega_{\text{axial}} - \gamma) \right] \tag{1.b}
\]

where \(\Omega_{\text{axial}} \text{ and } \gamma \text{ are as defined in the Appendix.}

On the other hand, the external axial work done by a unit axial load is given as:

\[
W_{\text{ext, axial}} = (u^1 - u^2). \tag{1.c}
\]

Equating the internal and external works:

\[
(u^1 - u^2) = \frac{f_1}{EA} \left[ 2a + Le + \frac{4}{n_{\text{axial}}} (2\Omega_{\text{axial}} - \gamma) \right]. \tag{1.d}
\]
For equilibrium, $f_x^1 = -f_x^2$. Therefore:

$$\begin{align*}
\{ f_x^1 \} = & \frac{EA}{L_{R,\text{axial}}} \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \{ a^1 \} \\
\end{align*} \quad (1.e)$$

where $L_{R,\text{axial}} = 2a + L_c + \frac{4}{n_{\text{axial}}} (2\Omega_{\text{axial}} - \gamma)$. In the limit where there is no flange reduction, $L_{R,\text{axial}}$ will reduce to $L$, the beam length.

### 2.2. Virtual work in torsional direction

The internal work done by a unit torsional load applied at end ‘1’ of a beam element with RBS ignoring torsional warping [9,23] is:

$$W_{\text{int,torsional}} = 2 \int_0^a \frac{m_4^1}{GJ} \mathrm{d}x + \int_0^{L_c} \frac{m_4^1}{GJ} \mathrm{d}x + 2 \int_{-b}^b \frac{m_1^1}{GJ} \mathrm{d}x$$

$$= \frac{m_4^1}{GJ} \left[ 2a + L_c + \frac{4}{n_J} (2\Omega_J - \gamma) \right] \quad (2.a)$$

Integration gives:

$$W_{\text{int,torsional}} = \frac{m_4^1}{GJ} \left[ 2a + L_c + \frac{4}{n_J} (2\Omega_J - \gamma) \right] \quad (2.b)$$

where $n_J = \frac{4a^3}{GJ}$, $G$ is the shear modulus, $J$ is the torsional constant, and $I_f$ is the top flange thickness. $\Omega_J$ and $\gamma$ are as defined in the Appendix. The joint force, $m_4^1$, represents a torsional moment and is shown in Fig. 2 along with the other joint forces.

On the other hand, the external work done by a unit torsional load is given as:

$$W_{\text{ext,torsional}} = \left( \theta_x^1 - \theta_x^2 \right). \quad (2.c)$$

Equating the internal and external works,

$$\left( \theta_x^1 - \theta_x^2 \right) = \frac{m_4^1}{GJ} \left[ 2a + L_c + \frac{4}{n_J} (2\Omega_J - \gamma) \right]. \quad (2.d)$$

For equilibrium, $m_4^1 = -m_3^2$. Therefore:

$$\begin{align*}
\{ m_3^1 \} = & \frac{GJ}{L_{R,J}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \{ \theta_x^1 \} \\
\{ m_3^2 \} = & \frac{GJ}{L_{R,J}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \{ \theta_x^2 \} \\
\end{align*} \quad (2.e)$$

where $L_{R,J} = 2a + L_c + \frac{4}{n_J} (2\Omega_J - \gamma)$. In the limit, $L_{R,J}$ will reduce to $L$, the beam length.

### 2.3. Virtual work in major axis bending

The internal virtual work done by $m_1^1$, moment at end ‘1’ and the moment due to $f_x^1$, shear at the same end of RBS beam is:

$$W_{\text{int,majorbending}} = \sum_{i=1}^{5} W_{\text{int,majorbending}}^i \quad (3.a)$$

where $i = 1, 5$ is the number of distinct geometries in the RBS beam.

$$\begin{align*}
W_{\text{int,majorbending}}^1 = & \frac{f_x^1}{EI_{\text{maj}}} \int_0^a -x \mathrm{d}x + \frac{m_1^1}{EI_{\text{maj}}} \int_0^a \mathrm{d}x \\
W_{\text{int,majorbending}}^2 = & \frac{f_x^1}{EI_{\text{maj}}} \int_0^a -x \mathrm{d}x + \frac{m_1^1}{EI_{\text{maj}}} \int_0^a \mathrm{d}x \\
W_{\text{int,majorbending}}^3 = & \frac{f_x^1}{EI_{\text{maj}}} \int_0^a -x \mathrm{d}x + \frac{m_1^1}{EI_{\text{maj}}} \int_0^a \mathrm{d}x \\
W_{\text{int,majorbending}}^4 = & \frac{f_x^1}{EI_{\text{maj}}} \int_0^a -x \mathrm{d}x + \frac{m_1^1}{EI_{\text{maj}}} \int_0^a \mathrm{d}x \\
W_{\text{int,majorbending}}^5 = & \frac{f_x^1}{EI_{\text{maj}}} \int_0^a -x \mathrm{d}x + \frac{m_1^1}{EI_{\text{maj}}} \int_0^a \mathrm{d}x.
\end{align*} \quad (3.b-f)$$

Adding all the five terms and integrating:

$$\begin{align*}
(W_{\text{int,majorbending}})_{\text{total}} = & -\frac{f_x^1}{EI_{\text{maj}}} L \left( a + \frac{L_c}{2} \right) + \frac{m_1^1}{EI_{\text{maj}}} (2a + L_c) \\
& - \frac{f_x^1}{EI_{\text{maj}}} L \left( n_{\text{maj,bend}} \left( \Omega_{\text{maj,bend}} - \frac{\gamma}{2} \right) + m_1^1 \left( n_{\text{maj,bend}} \left( \Omega_{\text{maj,bend}} - \gamma \right) \right) \quad (3.g)$$

where $I_{\text{maj}}$ is the major axis moment of inertia, ‘e’ and ‘f’ are as shown in Fig. 1. $\Omega_{\text{maj,bend}}, n_{\text{maj,bend}}$ and $\gamma$ are defined in the Appendix.

Now, equating external work to internal work done on RBS beam element due to major axis bending and simplifying the ensuing expression:

$$\begin{align*}
1.\theta_y^1 = & -\frac{S_{\text{maj}}}{EI_{\text{maj}}} f_x^1 + \frac{T_{\text{maj}}}{EI_{\text{maj}}} m_1^1 \quad (3.h)
\end{align*}$$

Please cite this article in press as: Kassegne SK. Development of a closed-form 3-D RBS beam finite element and associated case studies. Engineering Structures (2006), doi:10.1016/j.enganstruct.2006.09.010
where
\[ S_{\text{maj}} = L \left( a + \frac{L_e}{2} \right) + \frac{L}{\eta_{\text{maj,bend}}} \left( \Omega_{\text{maj,bend}} - \frac{\gamma}{2} \right) \]
\[ T_{\text{maj}} = 2a + L_e + \frac{1}{\eta_{\text{maj,bend}}} \left( 2\Omega_{\text{maj,bend}} - \gamma \right). \]

In the limit where there is no flange width reduction, \( S_{\text{maj}} \) and \( T_{\text{maj}} \) reduce to \( L^2/2 \) and \( L \) respectively.

2.4. Virtual work done in major axis shear including shear deformation

The virtual work done due to \( f_z^1 \), a unit shear force applied in the major axis direction at end ‘1’ of a beam with RBS connection is:
\[ W_{\text{int,majorshear}} = \sum_{i=1}^{5} W_{\text{int,majorshear}}^i \]  

(4.a)

where \( i = 1, 5 \) is the number of distinct geometries in the RBS beam.

\[ W_{\text{int,majorshear}}^1 = \frac{f_z^1}{E I_{\text{maj}}} \int_0^a x^2 dx - \frac{m_y^1}{E I_{\text{maj}}} \int_0^a x dx + \frac{f_z^1}{G A_{\text{maj}}} \int_0^a dx \]  

\[ W_{\text{int,majorshear}}^2 = \frac{f_z^1}{E I_{\text{maj}}} \int_0^b (e + x)^2 dx \]  

\[ - \frac{m_y^1}{E I_{\text{maj}}} \int_b^a \left[ 1 - 4\eta_{\text{maj,bend}}(\sqrt{R^2 - x^2} + R') \right] (e + x) dx \]  

\[ + \frac{f_z^1}{G A_{\text{maj}}} \int_b^a dx. \]  

(4.b)

\[ W_{\text{int,majorshear}}^3 = \frac{f_z^1}{E I_{\text{maj}}} \int_0^{L_e} (a + 2b + x)^2 dx \]  

\[ - \frac{m_y^1}{E I_{\text{maj}}} \int_0^{L_e} (a + 2b + x) dx + \frac{f_z^1}{G A_{\text{maj}}} \int_0^{L_e} dx \]  

\[ W_{\text{int,majorshear}}^4 = \frac{f_z^1}{E I_{\text{maj}}} \int_0^b (f + x)^2 dx \]  

\[ - \frac{m_y^1}{E I_{\text{maj}}} \int_b^a \left[ 1 - 4\eta_{\text{maj,bend}}(\sqrt{R^2 - x^2} + R') \right] (f + x) dx \]  

\[ + \frac{f_z^1}{G A_{\text{maj}}} \int_b^a dx. \]  

(4.d)

\[ W_{\text{int,majorshear}}^5 = \frac{f_z^1}{E I_{\text{maj}}} \int_0^a (L - a + x)^2 dx \]  

\[ - \frac{m_y^1}{E I_{\text{maj}}} \int_0^a (L - a + x) dx + \frac{f_z^1}{G A_{\text{maj}}} \int_0^a dx. \]  

(4.f)

Now, adding all the five terms and integrating,
\[ (W_{\text{int,majorshear}})_{\text{total}} = \frac{U_{\text{maj}}}{E I_{\text{maj}}} f_z^1 + \frac{H_{\text{maj}}}{G A_{\text{maj}}} \frac{f_z^1}{E I_{\text{maj}}} - \frac{S_{\text{maj}}}{E I_{\text{maj}}} m_y^1 \]  

(4.g)

where
\[ U_{\text{maj}} = \left[ \frac{2a^3}{3} + \frac{L_e^3}{3} + aL_e(L_e + a) + 2bL_e(L - 2b) \right] \]  

\[ + \frac{1}{4\eta_{\text{maj,bend}}} \left[ -2\gamma(2R^2 + e^2 + f^2) \right] \]  

\[ + 4\eta_{\text{maj,bend}} \left[ e^2 + f^2 + 2R^2 \left( 1 - \frac{1}{\alpha^2} \right) \right] + 2\alpha_{\text{maj,bend}} \]  

\[ H_{\text{maj}} = L \]  

\[ S_{\text{maj}} = L \left( a + \frac{L_e}{2} \right) + \frac{L}{\eta_{\text{maj,bend}}} \left( \Omega_{\text{maj,bend}} - \frac{\gamma}{2} \right). \]  

\[ \lambda \] is defined in the Appendix. In the limit, \( U_{\text{maj}} \) and \( S_{\text{maj}} \) will reduce to \( L^3/3 \) and \( L^2/2 \) respectively.

Now, equating external work to internal work done on RBS beam element due to major axis shear:
\[ 1.w^1 = \frac{U_{\text{maj}}}{E I_{\text{maj}}} + \frac{H_{\text{maj}}}{G A_{\text{maj}}} \frac{f_z^1}{E I_{\text{maj}}} - \frac{S_{\text{maj}}}{E I_{\text{maj}}} m_y^1. \]  

(4.h)

In matrix form, the equations of equilibrium, i.e. Eqs. (3.h) and (4.h) could be re-written together as:
\[ \begin{bmatrix} U_{\text{maj}} + H_{\text{maj}} \end{bmatrix} \begin{bmatrix} f_z^1 \\ m_y^1 \end{bmatrix} = \begin{bmatrix} S_{\text{maj}} \\ T_{\text{maj}} \end{bmatrix}. \]  

(4.i)

Inversion of the above matrix yields:
\[ \begin{bmatrix} f_z^1 \\ m_y^1 \end{bmatrix} = \left( U_{\text{maj}} + \Phi_{\text{maj}} \right) T_{\text{maj}} - S_{\text{maj}} S_{\text{maj}} \]  

\[ \times \begin{bmatrix} T_{\text{maj}} \\ S_{\text{maj}} \end{bmatrix} \begin{bmatrix} \frac{U_{\text{maj}}}{E I_{\text{maj}}} + \frac{H_{\text{maj}}}{G A_{\text{maj}}} \frac{f_z^1}{E I_{\text{maj}}} \\ \frac{S_{\text{maj}}}{E I_{\text{maj}}} \end{bmatrix} \]  

(4.j)

where \( \Phi_{\text{maj}} \), the major axis shear deformation term is given as:
\[ \Phi_{\text{maj}} = \frac{H_{\text{maj}} E I_{\text{maj}}}{G A_{\text{maj}}}. \]  

2.5. Virtual work in minor axis bending

The virtual work done in the minor axis is very much analogous to that of the major axis with the exception that the net minor axis moment of inertia (\( I_{\text{min}} \)) in the flange reduction area is calculated differently with the reductions assuming the third power of the removed flange width (see the Appendix and Fig. 1(b)). Now, following the same procedure as that of the major axis bending, the final expression for the work done in minor axis bending is of the form:
\[ (W_{\text{int,minorbending}})_{\text{total}} = -\frac{f_y^1}{E I_{\text{min}}} L \left( a + \frac{L_e}{2} \right) \]  

(4.k)
\[ + \frac{m_1^2}{EI_{\min}} (2a + L_e) \]
\[ - \frac{f_1^b}{EI_{\min}} \int_a^b \frac{(e + x) + (f - x)}{1 - \frac{4f}{m_1} \left( \frac{b_f - b_x}{2} \right)^2} dx \]
\[ + \frac{1}{m_1} \int_a^b 2dx \]  
(5.a)

where \(b_x = \sqrt{R^2 - x^2} + R\) (as given in the Appendix).

Now, equating external work to internal work done on RBS beam element due to minor axis bending and simplifying the ensuing expression gives:

\[ 1. \theta^e = -\frac{S_{\min}}{EI_{\min}} f_1 + \frac{T_{\min}}{EI_{\min}} m_1 \]  
(5.b)

where \(S_{\min}\) and \(T_{\min}\) can be evaluated through numerical quadrature as:

\[ S_{\min} = L \left( a + \frac{L_e}{2} \right) + \int_a^b 2dx \]
\[ T_{\min} = 2a + L_e + \int_a^b 2dx \]

In the limit where there is no flange width reduction, \(S_{\min}\) and \(T_{\min}\) will reduce to \(L^2/2\) and \(L\) respectively.

2.6. Virtual work in minor axis shear including shear deformation

The same procedures used for major axis shear case are repeated here for deriving the virtual work done, the end moments and end shear forces. Similar expressions for the minor axis bending and shear could be written by replacing the major axis properties with minor axis properties. However, it has to be noted that the minor shear area is not neglected in the minor axis direction unlike the major axis case and, therefore, additional shear terms will show up in the virtual work as shown below. For brevity, only the final expressions are given here taking into account the additional terms due to reduction in minor shear area.

\[ (W_{\text{int, minorshear}})_{\text{total}} = \frac{f_1^b}{EI_{\min}} \left[ \frac{2a^3}{3} + \frac{L_e^3}{3} \right] \]
\[ + aL_e(L_e + a) + 2bL_e(L - 2b) + a(L - a) \]
\[ + \frac{f_1^b}{GA_{x_{\min}}} \left[ 2a + L_e \right] - \frac{m_1^2}{EI_{\min}} \left( a + \frac{L_e}{2} \right) \]
\[ + \frac{f_1^b}{EI_{\min}} \int_a^b \frac{(e + x)^2 + (f + x)^2}{1 - \frac{4f}{m_1} \left( \frac{b_f - b_x}{2} \right)^2} dx \]
\[ + \frac{f_1^b}{GA_{x_{\min}}} \int_a^b 2dx \]  
\[ + \frac{1}{m_1} \int_a^b \frac{(e + x) + (f - x)}{1 - \frac{4f}{m_1} \left( \frac{b_f - b_x}{2} \right)^2} dx \]
\[ + \frac{1}{m_1} \int_a^b 2dx \]  
(6.a)

Simplifying,

\[ (W_{\text{int, minorshear}})_{\text{total}} = \frac{U_{\min}}{EI_{\min}} f_1^b + \frac{H_{\min}}{GA_{\min}} f_1^b - \frac{S_{\min}}{EI_{\min}} m_1^2 \]  
(6.a)

where \(S_{\min}\) is defined in Section 5 and \(H_{\min}\) is as given below:

\[ H_{\min} = 2a + L_e + \frac{4}{n_{\min, shear}}(2\Omega_{\min, shear} - \gamma) \]

\(U_{\min}\) can be evaluated through numerical quadrature as:

\[ U_{\min} = \left[ \frac{2a^3}{3} + \frac{L_e^3}{3} + aL_e(L_e + a) \right] \]
\[ + 2bL_e(L - 2b) + a(L - a) \]
\[ + \int_a^b \frac{(e + x)^2 + (f + x)^2}{1 - \frac{4f}{m_1} \left( \frac{b_f - b_x}{2} \right)^2} dx \]
\[ + \frac{1}{m_1} \int_a^b \frac{(e + x) + (f - x)}{1 - \frac{4f}{m_1} \left( \frac{b_f - b_x}{2} \right)^2} dx \]
\[ + \frac{1}{m_1} \int_a^b 2dx \]

\(n_{\min, shear}, \Omega_{\min, shear}\) and \(\gamma\) are defined in the Appendix. In the limit, \(U_{\min}\) and \(H_{\min}\) will reduce to \(L^3/3\) and \(L\), respectively.

Following the same procedure as in the major axis bending and shear, the stiffness matrix that relates the minor axis shear and moment at end ‘1’ with the corresponding transverse displacement and rotation at the same end is derived:

\[ \begin{bmatrix} f_1^b \\ m_1^2 \end{bmatrix} = \frac{EI_{\min}}{(U_{\min} + \Phi_{\min})T_{\min} - S_{\min}S_{\min}} \times \begin{bmatrix} T_{\min} \\ S_{\min}(U_{\min} + \Phi_{\min}) \end{bmatrix} \begin{bmatrix} u_1 \\ \Theta_{\min} \end{bmatrix} \]  
(6.c)

where \(\Phi_{\min}\), the minor axis shear deformation term is given as:

\[ \Phi_{\min} = \frac{H_{\min}E_{I_{\min}}}{GA_{\min}} \]

The displacement-joint force relationships for the release of joint ‘2’ are identical to the relationships for the release of joint ‘1’. These are represented as \([K_{22}]\) in Eq. (7.a).

\[ \begin{bmatrix} [K_{11}] & [K_{12}] \\ [\text{Symm.}] & [K_{22}] \end{bmatrix} \begin{bmatrix} \{A\}^1 \\ \{A\}^2 \end{bmatrix} = \begin{bmatrix} \{F\}^1 \\ \{F\}^2 \end{bmatrix} \]  
(7.a)

\[ m_2^2 = f_2^b L - m_1^2 \]  
(7.b)

where \(\{A\}^1\) and \(\{A\}^2\) are each arrays of six displacement components at joints ‘1’ and ‘2’ and \(\{F\}^1\) and \(\{F\}^2\) are arrays of the six joint force components at joints ‘1’ and ‘2’, respectively.

The final 12×12 stiffness matrix and stiffness equation based on Tiomoshenko’s Beam Theory – as shown in Box I –
for a beam element with RBS connections at both ends is, then, given by combining Eqs. (1.e), (2.e), (4.j), (6.c) and (7.b): see Box I where

\[ \begin{bmatrix} K_{11} & K_{22} & K_{33} & K_{35} & K_{44} & K_{55} & K_{66} \\ K_{22} & -K_{11} & -K_{26} & K_{26} & -K_{33} & -K_{44} & K_{55} \\ K_{33} & -K_{26} & -K_{35} & -K_{44} & K_{55} & -K_{55} & -K_{55} \\ K_{35} & -K_{26} & -K_{35} & -K_{55} & -K_{55} & -K_{55} & -K_{55} \\ K_{44} & K_{55} & K_{55} & K_{55} & K_{55} & K_{55} & K_{55} \\ K_{55} & -K_{44} & K_{55} & K_{55} & K_{55} & K_{55} & K_{55} \\ K_{66} & K_{55} & K_{55} & K_{55} & K_{55} & K_{55} & K_{55} \end{bmatrix} = \begin{bmatrix} u_1^1 \\ v_1^1 \\ \theta_z^1 \\ \theta_x^1 \\ \theta_y^1 \\ \theta_z^2 \\ \theta_x^2 \end{bmatrix} \]

\[ = \begin{bmatrix} f_1^1 \\ f_2^1 \\ m_1^1 \\ m_1^2 \\ m_2^1 \\ m_2^2 \end{bmatrix} \]

Box I.

The width of the radius cut, ‘c’ as shown in Fig. 1 is given by:

\[ c = 0.5 \times (1 - r) \times \text{BeamWidth} \] (8.a)

where ‘r’ varies from 0.5 to 0.9 (which correspond to 50% to 10% cut). In general, the percentage of flange removal is taken as (2c/beamwidth) = 100.

On the other hand, there are two sets of recommendations for the dimensions of the parameters ‘a’, and ‘b’ (Fig. 1). The recommendations of Moore et al. [16] and SAC [18] specify the following values:

\[ a = 50\%–75\% \text{ beam flange width}; \] (8.b)

\[ b = 65\%–85\% \text{ beam depth}. \] (8.c)

Iwankiw and Carter [14] recommend the following values:

\[ a = 25\%–50\% \text{ of the beam depth}; \] (8.d)

\[ b = 75\%–100\% \text{ of the beam depth}. \] (8.e)

In this study, information on which recommendation is based is provided for each example solved here. For simplicity, in all the examples reported in this study, the percentage reductions in RBS width are the same for all the beams in a given structure. The values of ‘a’ and ‘b’, of course, depend on the beam flange width and depth which are a function of the size of the ‘WF’ beam selected. In all the examples reported here, structural steel is assumed to have E (Young’s Modulus) of 29,000 ksi (2.0 \times 10^{11} \text{ Pa}) and Poisson’s ratio of 0.29.

3. Verification of 3D RBS finite element and example problems

The accuracy of the developed algorithm and program is tested using the following examples: cantilever and fixed beams with RBS connections, 2D frames with shear deformation under lateral loads, 3D frame with eccentric lateral loads, gravity loads and torsionally irregular framing, and 3D frames eigenvalue analysis. The quantities investigated are: effect on story and displacements, drift and mode shapes. Further, the increases in rotation stiffness coefficients (see Box I) in major and weak axis bending for some commonly used wide-angle beam sections are investigated.
shown in Fig. 3. The total value of the point loads is $-30 \text{kips}$ (133.4 kN) acting downward. The ANSYS model consists of 10-node tetrahedral structural solid (SOLID92 element) with about 11,500 elements for both models with and without RBS moment connections. The comparisons are given in Table 1 where the tip vertical displacements at the free end predicted by the RBS element developed in this study compare closely with those predicted by ANSYS.

The same cantilever was then subjected to a weak axis bending under a line of tip loads totaling 3 kips (Fig. 4). The mesh for the model with no RBS consisted of 18,195 tetrahedral structural solid elements whereas the mesh for the model with RBS connections had 18,416 elements. Table 1 summarizes the tip displacements obtained from a finite element analysis program [8] and the element developed in this study with both the new developed 3D element and FEA package giving increase in out-of-plane tip displacement in excess of 140%. This comparison demonstrates that the increases in weak axis displacements in the presence of RBS connections are considerable and much higher than those
corresponding to major axis displacements. Further, the same beam was reanalyzed with fixed end boundary conditions for both ends of the beam (Fig. 5). A lateral (weak axis) line load of 1 kip/ft (14.6 kN/m) was applied on both models with and without RBS connections. As summarized in Table 1, the FEA analysis using FEMLAB gives a close agreement with the present element in predicting the increase in weak axis bending; both showing about 36% increase in the center-span weak axis displacement due to the presence of RBS connections. These close comparisons in both major axis as well as minor axis bending between the new element and the commercial FEA packages [1,8] demonstrate the accuracy of the 3D RBS beam element developed here.

3.2. Drift in 6-story 2-bay 2D frame

This frame shown in Fig. 6 is very similar to the frame analyzed by Chambers [2] who reported a drift increase of 10.6% with a 40% flange-area reduction. Both column and beam sections are shown in the figure. The beam sizes vary from W24×68 at the roof to W30×235 in the bottom three stories whereas the top two floors have columns of size W24×279 and the bottom four floors have columns of size W24×370. The model solved by Chambers [2] is identical to this model except that the bottom four floors have columns of size W24×492 which is rarely used. The lateral loads are 85.71, 71.43, 57.14, 42.86, 28.57, 14.29 kips (1 kip = 4.448 kN) applied at the left ends of the frame starting from top floor and going down. The beams are free to deflect in the axial direction, as there is no diaphragm. A maximum reduction in flange width of 50% and $b = 0.5d$, and $a = 0.25d$ are used, as per Iwankiw and Carter [14] recommendations. Note that while the flange reductions in this example were varied from 10%–50% to demonstrate the increase in lateral displacements with different flange reductions, recommendations such as [18] limit the minimum flange reductions to 40%. The results are given in Table 2 which shows a maximum increase of 11.04% in lateral roof displacement using the current formulations. The

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small discrepancy between the two studies are due to shear deformation in the current formulation reported here and importantly – the use of different column sizes at the bottom four floors as discussed above.

3.3. Effect of RBS connections on beam stiffness coefficients

The quantitative assessment of the various stiffness coefficients for a beam element and their reduction in the presence of RBS connections is of practical interest. For a beam of W27×336 section with $I_{maj} = 14500$ in.4 ($6.035 \times 10^9$ mm4), $I_{min} = 1170$ in.4 (4.87 $\times 10^8$ mm4), $A = 98.7$ in.2 (63 677 mm2), $A_{maj} = 37.8$ in.2 (24 387 mm2), and $A_{min} = 55.27$ in.2 (35 658 mm2), for example, a detailed analysis is carried out to determine the magnitude of stiffness coefficients reductions in major axis and minor axis bending in the presence of RBS connections of various flange area cut ratio and beam length to depth ratios. The stiffness coefficients of practical interest are $K_{33}$ and $K_{55}$ given in Box I which represent the joint shear and rotational stiffnesses respectively for major axis bending. For minor axis bending, $K_{22}$ and $K_{66}$ given in Box I, which represent the joint minor shear and minor rotational stiffnesses respectively, are of interest.

Fig. 7 shows the percentage decrease of the major axis rotational and shear stiffness coefficients in the presence of RBS connections at both sides as compared to the full beam section stiffness coefficients. For $L/d$ of 10.0 and $a = 7.5$ in. (190.5 mm) and $b = 15.0$ (381.00 mm) which were determined using recommendation of Iwankiw and Carter [14], a maximum of about 15.5% reduction is observed for the shear stiffness coefficient ($K_{33}$), and 12.5% reduction for the rotational stiffness coefficient ($K_{55}$). As the length of the beam increases, these reductions in stiffnesses typically exhibit a decreasing trend. However, at small $L/d$ ratios (less than 10), the contribution from shear deformation becomes significant and it slows the overall reduction in stiffnesses due to the fact that there is no reduction in shear area in major axis bending as explained in the formulations section. For $L/d$ ratios below 8 or so – ratios quite uncommon in realistic frames – the shear deformations are so dominant that the stiffness reductions do actually start decreasing for decreased $L/d$ ratios. Fig. 7 shows this rather unexpected trend for low $L/d$ ratios.

For the same flange area reduction parameters, the minor axis bending results show a maximum of 67% reduction in shear stiffness coefficient ($K_{22}$) and 65% reduction for the rotational stiffnesses ($K_{66}$) as shown in Fig. 8. In the case of beams with RBS flange reductions only at the bottom flanges (as in retrofitting), these reductions will go down by half to about 33% and 32% respectively, assuming a negligible warping effect. These rather large minor axis stiffness coefficients reductions are due to the fact that the decrease in $I_{yy}$ (minor axis moment of inertia) in RBS beams is not linear (as in the case of major axis bending); but cubic in the presence of flange cuts as shown in the formulation section (see also the Appendix). A comparison of the reduction on the integrated minor and major axis inertias for a variety of wide-angle beam sizes is given in Fig. 11 which demonstrates that for W24×68 section, for example, the minor axis moment of inertia (integrated over the length of the radius-cut region) reduces by about 23% whereas the major axis moment of inertia shows a decrease of only 0.5% for $r = 0.5$. For deeper sections such as W36×150, the reductions in integrated moments of inertia are
down to 9% for minor axis and 0.4% for major axis for \( r = 0.5 \). For steel buildings with a composite floor deck, the floor provides a lateral support and a composite action to the beams whereby the actual weak axis bending may not be as much as predicted by this formulation. However, it has to be noted that the bottom flanges of these beams are still unsupported by the floor; making reduction in weak axis bending stiffness still an issue of concern. Further, for beams located at roofs which are typically made of light metal deck and beams in areas of slab opening where there is no substantial diaphragm support, the large reduction in weak axis bending stiffness could potentially be a design concern. Some of the adverse effects of the reduction in weak axis bending stiffness include high torsional moment requirement on supporting columns and also potential lateral buckling of such beams which is a function of the weak-axis stiffness of flanges in compression regions [6].

Figs. 7(b) and 8(b) show the effect of ratio of flange area reduction, \( r \), on the stiffness coefficients where an increase in \( r \) (hence less cut) gives a nonlinearly decreasing reduction in stiffnesses. Figs. 7 and 8 further compare the stiffness reductions obtained for the two commonly used recommendations of \([14,16]\) where it is shown that Moore et al. [16] give a slightly conservative results. Fig. 9 shows the effect of selection of the flange area reduction parameters. A 30% flange area cut and a narrower RBS region positioned further from the adjoining column at 15 in (381 mm) results in minor axis stiffness coefficient reduction of 45%, which is about 20% down from the case with a wider RBS region positioned nearer to columns. The effect of the presence of RBS moment connections on axial and torsional stiffness coefficients \( (K_{11}) \) and \( (K_{44}) \) is shown in Fig. 10 where the reductions are shown to be below 10% even for a 50% flange area reductions. In Fig. 11, the effect of compactness of a rolled beam section on the reduction on the integrated minor axis moment of inertia, \( I_{\text{min}} \), over the whole RBS region is demonstrated. The most

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which shows increases in drifts at the nodes increase by amounts varying from a maximum of 3% reduction in $I_{\text{min}}$ even for a 50% flange-area reduction.

### 3.4. Drift in 3-dimensional frame system with shallow columns

This is a 6-story building frame with non-symmetric frame layout as shown in Fig. 12. All the bays are 25 ft (7.62 m) wide with a uniform story height of 13 ft (3.962 m). For all frames, the columns sizes are W14×426 for all the top five stories and W14×455 for the ground floor. These sizes are pre-qualified as per the FEAM recommendations [7]. For frames 1–4, the beam sizes are W24×68 at the top story and W24×192 for all other stories. For frame 5, the beam sizes are W24×192 for all stories. Two lateral load cases are considered with 125, 100, 80, 60, 40, and 20 kips (1 kip = 4.448 kN) applied in both directions at location of 100 ft, 50 ft (30.50 m, 15.25 m). A uniform line load of gravity loads of 3 kips/ft (43.75 kN/m) is applied at all the beams in Frames 2 and 3. Radius cut of 50% based on Iwankiw and Carter recommendations [14] are used. In this building, the beams at each story level are constrained to move with the diaphragms; therefore there are no axial deformations and minor axis bending in the beams. In addition, full rigid end zones are considered.

The results are summarized in Fig. 13 which shows increases in drifts at each of the nodes in the building model in the presence of RBS for both lateral loads only and a combination of lateral and gravity loads in $x$- and $y$-directions. There are a total of 84 nodes with 14 nodes at each story as shown in Fig. 12. The nodes are numbered from top to bottom; nodes from 1 to 14 representing nodes in the roof. The figure shows that drifts at the nodes increase by amounts from a maximum of about 13% at the roof level to 7.5% at the second and first floors for a load combination of gravity and lateral loads in $x$- and $y$-directions. The effects of different radius cut ratios and bay widths is demonstrated in Fig. 14 which shows that the drift increases could go higher to 13.5% if the bay width is reduced from 25 ft (7.62 m) to 20 ft (6.096 m) for a 50% flange area reduction. For lower flange area reductions, the drift increases go down to 10% and less, as expected.

### 3.5. Drift in 3-dimensional unsymmetrical frame system with deep columns

This is a 6-story building frame with symmetric lateral frames in the $x$- and nonsymmetrical frames in the $y$-directions as shown in Fig. 15. Deep columns (W24s) are used in this example to assess the effect of RBS connections on such framing systems. The objective of this example is primarily to investigate if the increases in story drifts in frames with deep columns follow the same trends as in shallower columns. Incidentally, recent research argues that deep columns – which

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are not pre-qualified by present recommendations – behave very well inelastically [20,17] contrary to earlier research that had suggested instability failures [22]. The debate is expected to continue as results obtained from such research as this one already indicate substantial reductions in weak axis bending of beams with RBS connections which will put additional torsional requirements on the framing columns. In light of this, therefore, it is believed that a review of the elastic behaviour of such systems, particularly from drift perspective, has some merit.

In the x-direction, two 2-bay parallel frames (1 and 2) – with equal bay spacing of 24 f (7.315 m) – are used at the outer perimeter edge. In the y-direction, 2-bay frames (3 and 4) are used at the outer edge of the building perimeter with the same 24 f spacing. The building dimensions are 120 ft x 96 ft (36.58 m x 29.26 m) with a uniform story height of 13 ft (3.96 m). The elevation of Frames 1, 2 and 3 is given in Fig. 16 where beam sizes vary from W30x235 (bottom floor) to W24x68 (roof). All the columns in all the frames are W24x335. Frame 4 has identical column sizes as Frame 1, but all the beams are W24x68s; thus introducing stiffness irregularity.

In this SMRF, independent lateral loads in the x- and y-directions (with total base shear of 420 kips respectively) were applied. The x-direction lateral loads are (120, 100, 80, 60, 40, and 20 kips) (1 kip = 4.448 kN) applied at a location of 60 ft, 8 ft (28.29 m, 2.44 m). The same load pattern is applied in the y-direction as well at a location of 100 ft, 0 ft (30.5 m, 0 m). A uniform line load of gravity loads of 3 kips/ft (43.75 kN/m) is applied at all the beams in Frames 2 and 3. Further, an eigenvalue analysis is also carried out to determine the effect of RBS moment connections on the dynamic characteristics of the frame. Radius cuts of 10%, 40% and 50% were used. The radius-cut parameters are based on Iwankiw and Carter recommendations [14]. The beams at each story level are constrained to move with the diaphragms; therefore there are no axial deformations and minor axis bending in the beams. Full rigid end zones are considered.

The results are summarized in Figs. 17–19. There are a total of 66 nodes with 11 nodes at each story as shown in Fig. 13. The nodes are numbered from top to bottom; nodes from 1–11 representing nodes in the roof. For a load combination of dead load and x-direction lateral load case, Fig. 17 shows that a maximum of about 13.5% increase in drifts at the nodal points is observed for RBS connections with radius cut of 50%. The lateral loads by themselves result in about 13% drift increase at the roof level which drops to about 7% in the first floor. As shown in Fig. 19, in the presence of RBS moment connections, the y-direction loads cause even higher drift increases within the range of 15% at the roof level for a combination of gravity and lateral loads (1.2DL + 1.4E) and about 14.4% for a lateral load case. The drift increases are observed to decrease at the lower stories reaching drift values as low as 7%. The effect of beam sizes on the drift increases – while keeping column sizes unaltered – is shown in Fig. 18 where the deeper and stiffer W30x235 beams (I_maj = 11,700 in. 4 or 4.87 x 10^9 mm^4) are replaced by the less-stiff W24x250 wide-flange sections (I_maj = 8490 in. 4 or 3.53 x 10^9 mm^4). Dropping the beam sizes to W24x250, increases the drift due to x-direction earthquake load in the presence of RBS moment connections from 13% to about 15%. This increase is due to the fact that the contribution of beams towards the lateral displacements and drifts increases when smaller sizes are used. This trend of increased degradation of lateral stiffness in frames with large columns and smaller beam sizes with RBS moment connection is an important demonstration of the inadequacy of the ‘one-size-fits-all’ approach of 9% drift increase recommended by FEMA [7] and SAC [18].

The eigenvalue comparisons indicate a modest average increase of 6% for a 50% radius-cut. This smaller increase in periods is as expected because the period of a structure is directly proportional to the square root of the stiffness, which suggests that the increase will be increasing not linearly but at the slower rate of the square root of the decrease in stiffness.
Fig. 16. Elevation view of Frames 1, 2 and 3. Equal bay-widths and story heights.

Fig. 17. Effect of RBS connection on story displacements for lateral loads in x-direction and a combination of gravity and lateral loads.

Fig. 18. Effect of beam size on drift increases in x-direction for SMRF with RBS connections. Column sizes are kept constant and 1.2DL + 1.4E combination is used.

Fig. 19. Effect of RBS connection on story displacements for lateral loads in y-direction and a combination of gravity and lateral loads.

4. Conclusions

The study shows that the presence of RBS moment connections in SMRF could cause increase of story displacements as much as 15% in cases where diaphragm rotation is present under gravity and lateral loads. Approximate analysis tools based on 2-dimensional frames where there were no rotations had reported drifts of about 10%. The additional drifts reported in this study are, therefore, caused by consideration of shear deformations and diaphragm rotations—quantities not studied in prior research work. The effect of combining gravity and lateral loads—at least for cases reported here—are not substantial in terms of increasing story drifts. Results from this study also indicate that frames with both shallow and deep columns are sensitive to drift increase due to the introduction of RBS connections.

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Further, this study also highlights that the assumption that a 10% increase in story displacements translates to an equal amount of drift is erroneous since the effect of RBS connections at each of the story levels is different resulting in a potentially higher drift increases.

The study also predicts that the decrease in weak axis bending stiffness of beams with RBS connections on both sides – and unsupported laterally – could be as high as 67%. For beams subjected to weak axis bending where either the bottom flange or both flanges of these beams is still unsupported by the floor and for beams located at roofs which are typically made of light metal deck, this drastic decrease in weak axis bending stiffness could have adverse effects due to the large reduction in stiffness available to resist this bending. Some of these adverse effects include high torsional moment requirement on supporting columns and also potential lateral buckling of such beams in the laterally unsupported regions. Work in extending the current formulations to assessing elastic lateral torsional buckling (LTB) in beams with RBS moment connections is already under progress.

This study that takes into account shear deformation and effect of diaphragm rotations and gravity and lateral loads combination is considered more realistic and based on rational analysis and, through numerous examples reported here and compared with commercial FEA programmes, has been verified to give accurate results.

Therefore, it is recommended that the analysis and design of special moment resisting frames (SMRF) with RBS moment connections using the traditional 9% and 10% increases be reviewed in light of these results. With this new work, rational analysis based on realistic modelling of the member stiffness based on its reduced section is now available to structural engineers for modelling 3-dimensional frames with Timoshenko’s Beam Theory and its use should be encouraged for routine structural design practice.

Acknowledgements

The contribution of Young Li in using a mathematical software for symbolic manipulation of some of the integrals and Jeff Milton in independently verifying some of the results is gratefully acknowledged.

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[3] Chen SJ, Yeh CH. Enhancement of ductility of steel beam-to-column connections using the traditional 9% and 10% increases be reviewed in light of these results. With this new work, rational analysis based on realistic modelling of the member stiffness based on its reduced section is now available to structural engineers for modelling 3-dimensional frames with Timoshenko’s Beam Theory and its use should be encouraged for routine structural design practice.

Acknowledgements

The contribution of Young Li in using a mathematical software for symbolic manipulation of some of the integrals and Jeff Milton in independently verifying some of the results is gratefully acknowledged.

Appendix

\[
\gamma = \sin^{-1}\left(\frac{b}{R}\right)
\]
\[
\alpha_{\text{sub}} = \frac{R}{4R_{\text{sub}}}
\]
\[
\lambda_{\text{maj min}} = R^2 \left[ \gamma + \frac{\sin 2\gamma}{2} + \frac{2}{\alpha_{\text{maj min}} \sin \gamma + \alpha_{\text{maj min}}} \right]
\]
\[
n_{\text{maj bend}} = \frac{1}{n_{\text{maj}} \left[ \frac{b_1^2}{12} + \frac{t_f}{2} \left( \frac{d - t_f}{2} \right)^2 \right]}
\]
\[
n_{\text{min shear}} = \frac{4t_f}{A_{\text{min}} \left( \frac{5}{6} \right)}
\]
\[
n_{\text{min bend}} = \frac{1}{n_{\text{min}} \left[ \frac{b_1^2}{12} + \frac{t_f}{2} \left( \frac{b_f - b_2}{2} \right)^2 \right]}
\]

where \( b_x = \sqrt{R^2 - \alpha^2 + R^4} \).

The subscript ‘sub’ indicates that the expression is generic and is valid by replacing it with the appropriate term. For example, for axial stiffness, the subscript ‘sub’ is replaced by ‘axial’; for torsional stiffness, major axis bending stiffness, minor axis bending stiffness, major axis shear stiffness, minor axis shear stiffness, it is replaced by ‘J’, ‘major, bend’, ‘minor, bend’, ‘major, shear’, and ‘minor, shear’.

Uncited references

[3], [12], [13] and [21].

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